

**REPORT OF THE
MINISTER'S TASK FORCE
ON SENIOR HIGH SCHOOL
MATHEMATICS**

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EXECUTIVE SUMMARY

The Minister of Education appointed the Task Force on Senior High School Mathematics in February 2006 to consult widely and make recommendations on issues that had arisen during the ministry's review of the Grade 11 and 12 mathematics curriculum.

The task force involved a broad range of participants in its work. We interviewed more than 50 people from various walks of life – many outside the education sector – to get a better perspective on how what is taught in the mathematics classroom affects our society and economy.

We found enormous support for our efforts, leaving us more convinced than ever that the public cares deeply about what students learn in our schools.

Discussions with leaders in many different industries confirmed that mathematics is deeply embedded in today's workplaces. By studying mathematics, students not only learn useful techniques, they also learn to think critically and solve problems – key skills for success in the 21st century.

Through consultation, the task force also came to more fully appreciate the link between innovation and economic growth and to understand the connections between innovation, postsecondary education and the high school mathematics curriculum. Very quickly, the issue of the role of calculus in high school mathematics came to the fore.

The task force was repeatedly told that calculus is an essential vehicle for learning the problem-solving techniques demanded in today's sophisticated industries. We heard that our high schools must teach calculus if our students are to receive a solid grounding in this subject as the basis for success in mathematics-intensive university programs. It was stressed that, without access to calculus, Ontario students will fall behind their counterparts in other provinces and other countries.

These conversations convinced us that calculus belongs in the high school curriculum. But that left us with a challenge.

There is solid evidence that the performance of Ontario students as they begin university mathematics has suffered since the abolition of Grade 13 and the compression of five years of study into four. The key Grade 12 university-preparation mathematics course – *Advanced Functions and Introductory Calculus* – is overloaded with content, not leaving enough time to build reasoning and problem-solving skills. The ministry’s original proposals to revise the curriculum called for the removal of calculus so students could study other material in more depth. The task force supports the goal of a less dense Grade 12 university preparation course.

However, few people we interviewed supported the removal of calculus from the high school curriculum. We concurred and sought a way to retain calculus without compromising the learning of core aspects of mathematics. The solution we have endorsed is to introduce a new Grade 12 course that would include calculus as a fifth mathematics course for students preparing for mathematics-intensive university programs. At the same time, all students would be offered a Grade 12 mathematics course that excludes calculus and deals in depth with other important subject matter.

We realized it would not be feasible to simply add an extra course and tell schools to deliver it. A new course needs teachers, classrooms, textbooks and students. The only realistic way to bring in a new course is to replace an existing one. It so happens that the Grade 12 curriculum includes a mathematics course that has positive features but for a variety of reasons faces declining enrolment. Known as *Geometry and Discrete Mathematics*, this course in our view is unsustainable.

To replace it, the task force proposes to establish the new calculus course. But we do not feel the best approach is to devote this new offering entirely to calculus. *Geometry and Discrete Mathematics* contains a valuable component on vectors, which is useful preparation for engineering and physics among other fields. The task force proposes to retain the study of vectors by combining it with calculus in a new course to be called *Calculus and Vectors*. Both these subjects are taught in first-year university courses. In our view, the best preparation for Ontario students is to provide an exposure to both areas and let more depth be acquired in university.

The focus of this report is on the university preparation stream because this is where the main issues arose in the ministry's curriculum review process. The task force is satisfied that the ministry's proposals for curriculum revision in both the college and workplace streams should proceed essentially as currently planned. We also recommend keeping the existing Grade 12 university preparation course *Mathematics of Data Management* essentially as is, while improving its delivery.

The time for implementation of our recommendations is short. The revised mathematics curriculum for Grade 12 must be in place by September 2007 for students who will enter postsecondary programs in September 2008. The task force urges universities and colleges to begin work by September 2006 on their planning for revised entrance requirements. To facilitate this planning, the ministry should publicly release information about the revised Grade 12 mathematics curriculum by early September 2006 at the latest. This information could take the form of a broad course outline, while the ministry proceeds with writing the detailed curriculum in a parallel process.

In the latest Programme for International Student Assessment (PISA) study, Ontario Grade 9 students scored lower in mathematics than their counterparts in three other provinces. And as noted, the mathematics performance of first-year Ontario university students has declined. These trends are unacceptable, but fixing them will take more than the short-term measures we propose.

Many of the individuals we interviewed felt that options for revising the Grade 12 curriculum were limited by the fact that changes for the earlier grades had already been decided. It is the view of the task force that the entire mathematics curriculum, from Kindergarten through Grade 12, should be reviewed as a whole. If this is not feasible, then at least the Grades 7-12 curriculum should be reassessed as a unit.

The task force heard time and again that the latest round of curriculum review has not provided adequate opportunity for public engagement. To oversee the new comprehensive process, we recommend that the ministry establish a mathematics curriculum council representing a wide range of stakeholders. Teachers, school boards, postsecondary

programs, industry sectors, students and parents should all be included. This review should begin immediately, rather than waiting for the next round of a five-year cycle.

Improved teacher education – both pre-service and through professional development – is also an essential long-term strategy for improving student success in learning mathematics. We must strengthen teachers’ understanding of mathematics content as well as their grasp of instructional strategies, leading to better results in the classroom.

The task force is confident our recommendations will help achieve a mathematics curriculum that promotes success for all students, while meeting the needs of future leaders in the knowledge-based industries that drive economic growth.

ABOUT THE MINISTER'S TASK FORCE ON SENIOR HIGH SCHOOL MATHEMATICS

Chair:

Thomas Salisbury Professor of Mathematics and Statistics at York University, Deputy Director of the Fields Institute, incoming President of the Canadian Mathematical Society.

Members:

André Ladouceur Retired Head of Mathematics at Collège Catholique Samuel-Genest, Ottawa, Past President of Association française pour l'enseignement des mathématiques en Ontario (AFEMO), 1995 winner of the Prime Minister's Award for Excellence in Teaching in Science, Mathematics and Technology.

Husein Panju Grade 12 student at Richmond Hill High School and Student Trustee for the York Region District School Board.

Beverly Farahani Director with the Ontario Association for Mathematics Education and Head of Mathematics at Kingston Collegiate and Vocational Institute.

Manon Lemonde Past President of Parents partenaires en éducation, Secretary of the School Council at l'Ecole St-Charles Garnier in Whitby, former Chairperson of the School Council at l'Ecole St-Denis in Sudbury.

Consultant:

Wendy Hayes Managing Director for Apple Canada.

LIST OF RECOMMENDATIONS

The Minister's Task Force on Senior High School Mathematics recommends:

1. That calculus be retained in the Ontario high school mathematics curriculum, to prepare Ontario students for mathematically intensive fields of study at the university level in Ontario and preserve their access to such programs outside the province.
2. That in order to make room to provide students with a more solid mathematical foundation, calculus be removed from the Grade 12 *Advanced Functions and Introductory Calculus* course, and the course be renamed *Advanced Functions*.
3. That the course *Geometry and Discrete Mathematics* be eliminated, and replaced in the curriculum by a course tentatively named *Calculus and Vectors*. The new course would have *Advanced Functions* as a prerequisite or co-requisite.
4. That the content and evaluation of *Calculus and Vectors* be designed in such a way that good students are challenged and stimulated, but still have a reasonable chance of success in the course with a reasonable amount of effort.
5. That further consultations take place about what material should replace “rates of change” in *Advanced Functions*, and that consideration be given to moving topics between Grade 11 university preparation *Functions* and Grade 12 *Advanced Functions* in order to better balance both courses.
6. That the course *Mathematics of Data Management* be retained in essentially its current format.
7. That the ministry appoint an ongoing statistical advisory group, with representation from teachers and from statisticians (both academic and practising), to bring forward recommendations for improving the delivery of *Mathematics of Data Management*.

8. That the Grade 12 courses *Mathematics for Work and Everyday Life*, *Foundations for College Mathematics* and *Mathematics for College Technology* be implemented essentially as currently planned.
9. That the ministry release information about revised Grade 12 mathematics courses in two parallel processes, no later than September 5, 2006. One process would be the normal confidential review of the detailed curriculum proposals. The other would be a public release of short course summaries. On the basis of these summaries, universities and colleges could start to evaluate their entrance requirements and prerequisite structure.
10. That academic vice-presidents and deans of postsecondary institutions ensure that committees are formed prior to September 2006, both to review entrance requirements and prerequisites and to determine how courses should be changed in response to the revised high school curriculum.
11. That academic vice-presidents and deans consider the possibility of coordinated decisions about entrance requirements for specific programs (e.g., engineering) and plan timelines in such a way as to allow consultations on common requirements between universities.
12. That mathematically intensive programs where some but not all entering students will have taken *Calculus and Vectors*, universities should consider how to provide additional courses or additional support for students who have not taken this course.
13. That if *Calculus and Vectors* is an entrance requirement for a university program, the university involved should put in place mechanisms to ensure that students are not denied entry simply because the required course was not available to them.
14. That the ministry take steps to ensure that courses required or recommended for entry into programs at a substantial number of universities or colleges are available to all qualified Ontario students. In particular, financial support should be provided to schools where demand is low to allow them to offer such courses to smaller groups of

students than normally permitted. This is especially critical in the first year the revised curriculum is implemented.

15. That academic vice-presidents and deans make clear and public commitments to ensuring that the content of university courses is appropriate and accessible to students possessing the knowledge and skills laid out in the high school curriculum.
16. That a thorough evaluation of the entire Grade 7 to Grade 12 mathematics curriculum as a whole be carried out.
17. That the next review of the mathematics curriculum be done at a slower pace and in a more open manner, including the publication of "white papers" to ensure comprehensive and considered feedback and debate.
18. That the ministry provide support for the implementation of the Grades 7-12 mathematics curriculum in aboriginal communities.
19. That the ministry establish a mathematics curriculum council prior to September 2006 that would include representation from teachers, students, school boards, parents, industry sectors and relevant university and college disciplines.
20. That the mathematics curriculum council sponsor at least one open meeting each year, where teachers and representatives of postsecondary institutions would have an opportunity to discuss issues arising out of the high school curriculum.
21. That the ministry explore ways to allow students to more easily take two mathematics courses in Grade 11.
22. That greater attention be paid to the particular demands of teaching mathematics, through teacher recruitment, pre-service education and in-service education.

INTRODUCTION – MATHEMATICS IN THE KNOWLEDGE ECONOMY

The Minister of Education appointed the Task Force on Senior High School Mathematics in February 2006 with a mandate to consult widely about the mathematical knowledge and skills required by Ontario students to pursue postsecondary education or enter the workforce – and to make recommendations on directions for change in Ontario’s senior high school mathematics curriculum.

The members brought to the task force diverse experience as students, parents and teachers of mathematics at the secondary and postsecondary levels. We reviewed 24 written submissions and conducted 36 interviews with more than 50 individuals from a wide variety of college and university programs and economic sectors. We talked to university administrators or faculty in engineering, business, commerce, physics, education and mathematics; to college administrators; to executives or senior staff in sectors of the economy such as high technology, research and development, engineering, health care, pharmaceuticals, retailing, banking, law, mining and resources, construction, automobile assembly and other manufacturing; to educators in other provinces; to representatives of the aboriginal community; and to coordinators, teachers and parents.

The Global Context

The work of the task force took place in the context of a growing international emphasis on the connection between education and economic prosperity. The rise of a global, knowledge-based economy has given these issues a sense of urgency.

In the United States, for example, a report¹ from the National Academies Committee on Science, Engineering, and Public Policy recently observed: “Because other nations have, and probably will continue to have, the competitive advantage of a low-wage structure, the United States must compete by optimizing its knowledge-based resources, particularly in science and technology....” The report’s first recommendation for the United States to

¹ Committee on Science, Engineering, and Public Policy. *Rising Above the Gathering Storm – Energizing and Employing America for a Brighter Economic Future. Executive Summary.* Prepublication copy (2006), pp. 4, 6.

successfully compete in the 21st century is to “increase America’s talent pool by vastly improving K-12 science and mathematics education.”

A similar viewpoint is gaining ground in Europe. A Lisbon Council Policy Brief² comments: “Today, countries like China and India are starting to deliver high skills at low costs – and at an ever increasing pace....The challenge for Europe is clear. But so is the solution: evidence shows – consistently, and over time – that countries and continents that invest heavily in education and skills benefit economically and socially from that choice.”

The same could be said of Canada and Ontario – and many people are saying it. For example, Ontario’s colleges have completed a provincewide consultation called Pathway to Prosperity³. More than 600 employers, unions and other organizations from various economic sectors expressed concerns about Canada’s ability to meet the challenges of globalization, rapidly changing technology and an aging workforce. The first of five priorities for action highlighted in the report on these consultations is “relevant skills”, with both a higher level of skills and a greater number of people with skills viewed as necessary.

These broad themes emerged repeatedly during the interviews the task force held with a cross-section of industry sectors. We were told, and we fully agree, that Ontario cannot compete with low-wage economies on the basis of production costs. Our future prosperity depends on innovation to produce higher-value goods and services.

Engineering representatives and high-technology leaders conveyed a clear message that the Ontario economy depends on a cutting-edge and highly skilled workforce, with the capacity to engage in research and development. From designing automobiles, to developing wireless devices, to performing basic scientific research, mathematics is a prerequisite.

But we also learned that mathematics matters, not only to engineering and research, but also to an enormous range of jobs in the modern economy. We were told pharmacists

² A. Schleicher. *The Lisbon Council Policy Brief – The economics of knowledge: Why education is key for Europe’s success* (2006), p.2.

³ *Pathway to Prosperity – What We Heard* (2006), pp. 5-8.

need math to adjust medication dosages for age and weight....retail managers need math to align staffing schedules with the flow of customers....corporate executives need math to understand income statements and balance sheets....manufacturing employees need math to run equipment and maintain quality control....lawyers need math to use statistical evidence and prepare litigation strategies....construction workers need math to measure areas and calculate volumes.

More important than specific content is the thinking process that underlies mathematics. By studying mathematics, students learn how to reason logically, think critically and solve problems – key skills for success in today’s workplaces.

Demands on Today’s Mathematics Curriculum

Faced with these realities, Ontario’s mathematics curriculum must serve many diverse purposes. It must engage all students in mathematics and equip them to thrive in a society where mathematics increasingly permeates the workplace. It must engage and motivate as broad a group of students as possible, because early abandonment of the study of mathematics cuts students off from many career paths and postsecondary options. It must provide an adequate number of students with a solid grounding in the skills needed to enter mathematically intensive fields of study critical to the growth of the Ontario economy (such as engineering, science and business). It must nurture and stimulate those students interested in and capable of high mathematical achievement. It must preserve the accessibility of Ontario students to university programs outside the province. Planning a curriculum to meet these diverse needs is a complex task.

Curriculum under Review

The Ministry of Education reviews the curriculum for Ontario’s publicly funded schools through an ongoing, five-year cycle to ensure it remains relevant and continues to prepare students for future success. The review of the K-12 mathematics curriculum began in September 2003. The revised curriculum documents for Grades 9-10 and Grades 1-8 were implemented in September 2005.

The review of the Grades 11-12 mathematics curriculum included extensive research and analysis as well as consultation with teachers, universities, colleges, employers, parents,

students and others. The ministry completed the draft revised curriculum in summer 2005 and submitted it for feedback consultation in fall 2005, with implementation planned for September 2006. Much feedback was received on the proposals, both positive and negative.

The draft curriculum for Grades 11 and 12 was widely acknowledged to have dealt effectively with several key issues that arose from the abolition of Grade 13 and the need to compress five years of high school mathematics into four. For example, the draft curriculum proposed to:

- reduce overloading by shifting some topics to different courses, better balancing course content and removing some expectations entirely;
- address the high failure rate in the Grade 11 university/college preparation course by reducing overlap with the Grade 11 university preparation course; and
- create a clear pathway from Grades 9 and 10 in the applied stream to the Grade 12 mathematics courses preparing students for study at the college level.

Some recommendations proved more contentious, however. Significant objections were raised to the proposed removal of calculus from the Grade 12 *Advanced Functions and Introductory Calculus* university preparation course. Strong concerns were also expressed with an implementation schedule that left universities and colleges with little time to revise entrance requirements before students entering Grade 12 needed to select their courses.

Task Force Established

In response, the Minister of Education announced that the proposed changes to the Grade 11 curriculum would go ahead in September 2006, but the implementation of Grade 12 changes would be postponed until September 2007. The minister also appointed this task force to consult extensively and bring forward directions for change to the senior high school mathematics curriculum.

The goal of the task force has been to develop recommendations that will help all students succeed in their chosen destinations and also to meet the needs of students entering mathematics-intensive fields like engineering, science and finance. We are well aware that it is already difficult for employers in some sectors to find highly skilled employees.

Maintaining a high level of mathematical training, while at the same time seeking to interest and engage more students in mathematics, is essential for Ontario's future.

As suggested above, many of the proposed curriculum changes were viewed positively or were uncontroversial. Accordingly, they played a minor role in the consultations and discussions of the task force. This report focuses on the issues about which new decisions must be made. Several important questions are not examined here, simply because there is already broad agreement about how to handle them. For example, the proposed changes to workplace and college preparation courses received a largely positive response during the ministry's feedback process, and the task force has no new recommendations to make about the content of these courses.

It should be noted that the task force did not examine course content at the same level of detail as the ministry's curriculum review process did. Therefore it remains essential for the ministry to consider the results of the earlier research and feedback in designing the revised curriculum.

How the Report Is Organized

We begin the report by addressing the central question before us – the role of calculus in the high school mathematics curriculum.

Then we proceed to review the strengths and weaknesses in the current Grade 12 curriculum and make recommendations to address the problems in the short term.

Next, we turn to implementation challenges including a timetable for action.

Finally, we outline proposals for longer-term measures to improve the learning of mathematics in Ontario.

THE ROLE OF CALCULUS IN THE MATHEMATICS CURRICULUM

Today's global economic challenges demand both advanced mathematical knowledge and advanced problem-solving skills. This point takes us directly to what is no doubt the central issue before us – the role of calculus in the high school mathematics curriculum.

The task force heard repeatedly that calculus is an essential vehicle for learning the problem-solving techniques demanded by current economic realities. Even in fields that do not apply calculus concepts directly, the study of calculus is valuable because it requires complex problem-solving. The task force believes it is imperative to include calculus in the high school curriculum to give Ontario students an early grounding in this critical subject. This view is widely, almost universally, shared. Very few people we interviewed supported the removal of calculus from the high school curriculum.

Without calculus delivered in high school, the task force was warned that Ontario will fall behind other provinces and other countries. Without calculus, our graduates will not be well prepared for mathematics studies at the postsecondary level. Either universities will be forced to divert time and resources to help students catch up, or else stand aside and watch student performance decline. Neither makes sense in a competitive world. We also heard from some parents that concerns over mathematics preparation may push some students into private tutoring or private schools. Moreover, eliminating calculus would limit the access of Ontario students to mathematically intensive university programs in other jurisdictions in Canada and elsewhere.

Ontario students deserve a rich and innovative high school mathematics curriculum, one that is as at least as challenging as that of any other province maintaining a K-12 educational system.⁴ The task force believes the proposals given below meet this standard, and will provide sound preparation for postsecondary study. We also believe that students completing calculus according to the curriculum we are proposing will receive a better foundation in the subject than they receive at present.

⁴ All provinces but Quebec have a K-12 elementary and secondary school system, followed by postsecondary education. Quebec has six years of elementary school and five years of secondary school, followed by two years in general and vocational college (CEGEP) prior to university admission.

Are the proposals the best possible? In the short term, we believe they represent the best compromise available. But we also believe that a systematic examination of the entire K-12 mathematics curriculum, coupled with improved teacher education, can do better in the long term.

The current Grade 12 mathematics courses were offered for the first time in 2002-03, which was also the last year for the old Ontario Academic Courses (OAC). Will the proposals restore mathematics education to the levels reached in the OAC curriculum? No, because it is not realistic to expect students to learn in four years what they used to learn in five. The task force is convinced that pressure to compress as much as possible into four years has left many Ontario students with too superficial an understanding of both functions and calculus. In our view, the current mathematics curriculum does a disservice to many students. This is the reason we advocate immediate change to the Grade 12 curriculum, effective in September 2007, instead of waiting for longer-term reforms to provide gradual improvement.

Recommendation:

1. That calculus be retained in the Ontario high school mathematics curriculum, to prepare Ontario students for mathematically intensive fields of study at the university level in Ontario and preserve their access to such programs outside the province.

STRENGTHS AND WEAKNESSES IN GRADE 12 MATHEMATICS – AND PROPOSALS FOR CHANGE

Having concluded that calculus should be retained in the high school curriculum, we turn to the question of how to achieve this while also delivering solid preparation in other core areas of mathematics.

Assessing How Students Are Doing

One question facing the task force was whether the mathematical skills and preparation of Ontario students have fallen since the OACs were eliminated. The main issue is the

effectiveness of the current Grade 12 course on *Advanced Functions and Introductory Calculus* that is taken by most students preparing for university mathematics.

Evidence was presented that performance did not fall in the double cohort year itself – 2003-04 – but did drop afterwards. The Grade 12 portion of the double cohort was seen as exceptionally highly motivated, with exceptionally well developed study skills. This is not surprising given the threat they had faced since Grade 9 of unequal competition with the OAC cohort for postsecondary admission.

So let us put the double cohort year aside. Doing so leaves only two years of students who have gone through the current curriculum. While it may be premature to draw definitive conclusions, we do not have the luxury of waiting for more data if the curriculum is not meeting the needs of Ontario students.

Several universities have observed significant declines in student performance. Mathematics instructors generally report that students have less facility with the basic algebra, trigonometry, exponentials and logarithms required for mathematical study at the university level, including the study of calculus. Classes proceed more slowly because of a need to complete more algebraic steps in class. Knowledge of trigonometry is significantly eroded, since it is currently taught in Grade 11 but not in Grade 12. A number of universities report drops in test results, and drops in performance of Ontario students relative to those from some other provinces. It appears that Ontario students entering university mathematics now perform slightly below those from Alberta and significantly below those from Quebec, although comparisons with Quebec are difficult because its educational system is structured differently, as noted earlier.

This view was not unanimous, however. Some engineering and other university programs, which require two current Grade 12 mathematics courses for admission, reported either no drop in skills or a modest one. The required courses are *Advanced Functions and Introductory Calculus* and *Geometry and Discrete Mathematics*. This indicates that students who complete both of these courses have comparable preparation to their OAC counterparts. This positive picture could mean that this combination of courses prepares

students well for future study, or that only students with good mathematical skills actually take both courses.

At any rate, getting more students to take *Geometry and Discrete Mathematics* does not appear to be a viable option. Fewer students complete *Geometry and Discrete Mathematics* than was the case for OAC *Algebra and Geometry*, and that number is steadily falling. Also, the number of students doing three Grade 12 mathematics courses is far lower than the number who did three OAC mathematics courses.

Partly in response to falling enrolment, many university programs – including most Ontario engineering programs – are eliminating *Geometry and Discrete Mathematics* as a requirement for students entering in September 2006. We can only speculate whether those programs will continue to find their first-year students adequately prepared after this change.

The Problem with *Advanced Functions and Introductory Calculus*

The problem universities are finding is not a lack of specific knowledge that has been dropped from the high school curriculum. Rather, many students are no longer mastering basic skills and concepts that are still in the curriculum. They then struggle with this basic material in their first year in university. What the evidence suggests is that, as might have been expected, moving from five years of high school to four years has had a measurable impact on the mathematical preparation of Ontario students.

It is not only universities who have reported difficulties with the *Advanced Functions and Introductory Calculus* course – so have teachers and students. A common observation is that the course is overloaded with topics, containing much more material than was found in the former Grade 12 *Advanced Mathematics* course. While some topics were dropped when the OACs disappeared, it is also clear that substantially more than four years of the old OAC curriculum has been compressed into the four courses leading up to and including *Advanced Functions and Introductory Calculus*.

This course is frequently described as going too far too fast, with insufficient time to learn the topics deeply or to engage in open-ended investigation. This theme emerged

consistently in focus groups the ministry conducted with school boards, and in many of the ministry's consultations with parents, students, principals and teachers. At the same time, there is virtually a consensus that more effort should be devoted to trigonometry in high school. This means putting this subject back into Grade 12 to build on what has been learned in Grade 11. The ministry's proposals for curriculum revision addressed both the overloading and the trigonometry issues.

The task force heard from many sources that the precise list of topics covered matters less than developing solid skills in reasoning and problem-solving. It is more important for courses to have depth and focus on problem-solving than to include specific algorithms or techniques. What students need is a solid base to build on. Without that base, they won't succeed at university mathematics regardless of the specific techniques learned.

In fact, university mathematics departments generally voiced a preference for scaling back the list of topics students study in high school if this leads to a more solid understanding of what they do learn. While this would mean that mathematics departments may have to start teaching some topics from scratch, the alternative is superficial preparation that forces departments to re-teach large amounts of more elementary material. This point of view is consistent with the Western and Northern Canadian Protocol – a common mathematics curriculum being developed by the western provinces and the territories.

Again, there is a contrary viewpoint. We heard that in some schools with high province wide (EQAO) test scores and teachers having advanced mathematical training, the current *Advanced Functions and Introductory Calculus* course is working well. But on balance, the evidence from other high schools and from most mathematically intensive university programs seems to be otherwise. The task force is convinced that these students and programs would be better served if Grade 12 provided a more solid foundation in basic algebraic skills and a sounder understanding of fundamental functions (that is, trigonometry, exponentials and logarithms).

Impact of Grade 11 Curriculum Changes

It is possible that some of the changes to Grade 11 mathematics that are proceeding in September 2006 may eventually lead to some improvement in student results in Grade 12.

In particular, students needing more time to prepare for the study of calculus will now have the opportunity to take Grade 11 *Functions and Applications* followed by Grade 11 *Functions*, before going on to Grade 12. Alternatively, students can enter *Advanced Functions and Introductory Calculus* through *Functions and Applications* followed by Grade 12 *Mathematics for College Technology*.

It is tempting to wait a few years to see if changes like these can rescue the *Advanced Functions and Introductory Calculus* course. A complicating factor is the need to reinstate trigonometry in the Grade 12 curriculum, which would further compress this course. On balance, we do not believe that the existing course, plus trigonometry, will serve the needs of the majority of students without substantial change to the rest of the K-12 curriculum. That will not happen for some time, and every year we delay disadvantages another cohort of students. For this reason we advocate change now.

Recommendation:

2. That in order to make room to provide students with a more solid mathematical foundation, calculus be removed from the Grade 12 *Advanced Functions and Introductory Calculus* course, and the course be renamed *Advanced Functions*.

The Fate of *Geometry and Discrete Mathematics*

Geometry and Discrete Mathematics is a stimulating course for students having excellent mathematical skills and planning to study mathematics or computer science at the university level. It is increasingly avoided by other students, who would need to spend so much time on it that their ability to cope with other courses would be affected. Students preparing for university admission are conscious of the need for good grades, and worry that taking this course will drop their average. Also, the content of the course is not perceived by students heading for non-science programs as relevant to their plans. Even students interested in science and engineering do not generally see the course as reflecting the skills necessary for success in those programs. This is the case even though it includes problem-solving with vectors, a key tool in engineering and physics.

For all these reasons, enrolment in *Geometry and Discrete Mathematics* has declined to the point where smaller schools and rural boards are finding it hard to offer it. Many no

longer do. And the downward trend is likely to continue now that most university engineering programs have decided not to require this course in the future. The task force has been convinced – reluctantly – that the current *Geometry and Discrete Mathematics* course is unsustainable and should be dropped from the curriculum.

However, the task force believes that the vectors component of *Geometry and Discrete Mathematics* should be preserved and enhanced. Vectors is the topic that led engineering programs, until recently, to require this course for admission. The study of vectors is also useful preparation for other science and technology-based fields.

A Fifth Course on *Calculus and Vectors*

The removal of *Geometry and Discrete Mathematics* would create an opening for a new course in the Grade 12 mathematics curriculum, and it is here that we propose to place the study of calculus. Calculus would be combined with vectors in a new course tentatively named, as one might expect, *Calculus and Vectors*.

The new course would have *Advanced Functions* as a prerequisite or co-requisite and would therefore be a fifth high school mathematics course for students who select it.

Recommendation:

3. That the course *Geometry and Discrete Mathematics* be eliminated, and replaced in the curriculum by a course tentatively named *Calculus and Vectors*. The new course would have *Advanced Functions* as a prerequisite or co-requisite.

We heard proposals like this from several organizations, such as the Council of Ontario Universities and the Thames Valley District School Board. We discussed the idea with employers from various economic sectors, and in most cases heard support for this approach.

The downside is that certain aspects of proof and of discrete mathematics would no longer be offered to our best students in the regular curriculum. But this seems a fair trade-off for a deeper and more thorough grounding in essential mathematical skills for a larger group of students, including prospective scientists and engineers. At the same time, creating a curriculum less packed with topics opens up the possibility of providing supplemental or

enrichment material to the best students. This could include topics from the *Geometry and Discrete Mathematics* course or from the Advanced Placement (AP) or International Baccalaureate (IB) curriculums.

In this new fifth course, calculus could be treated in at least as much depth as in the current *Advanced Functions and Introductory Calculus*. In fact, since the students enrolling in the course will tend to be those with more advanced mathematics skills, the material may in fact be taught at a higher level than at present. Part of the problem with *Advanced Functions and Introductory Calculus* is that it must be delivered to a very broad range of students – much broader than is the case with calculus programs in other provinces. As a fifth high school mathematics course, a calculus course will be a better fit for its audience.

This recommendation would bring Ontario in line with most other provinces, where calculus is included in the fifth high school mathematics course. Quebec is the exception to this pattern – students in general and vocational colleges (CEGEPs) often take one or two half-courses in calculus. In the United States, calculus is offered in the high school curriculum or through AP or IB courses.

Topic order would need to be carefully specified so *Advanced Functions* could serve as a co-requisite for *Calculus and Vectors* in non-semestered schools. Including vectors as the first topic in the latter course would make it more feasible for non-semestered schools to run both courses concurrently.

The need for good grades is a reality for students. While *Calculus and Vectors* should stimulate and engage good mathematics students, care must be taken that it does not develop a reputation like *Geometry and Discrete Mathematics* had as a “killer course” – or one where success is possible only with disproportionate effort or at the expense of other courses.

Recommendation:

4. That the content and evaluation of *Calculus and Vectors* be designed in such a way that good students are challenged and stimulated, but still have a reasonable chance of success in the course with a reasonable amount of effort.

Why Not a Full Calculus Course?

The task force does not recommend offering a full course in calculus alone, for several reasons.

One is that there is very little geometric content in the senior high school curriculum – in either the current Grade 11 university preparation course (or the revised version to be implemented in September 2006) or in Grade 12 *Advanced Functions and Introductory Calculus*. The only way to study vectors is to take *Geometry and Discrete Mathematics*, and few students do. Combining vectors with calculus should provide a course with significant content that is both algebraic and geometric. Both areas are of value.

Engineering programs in Ontario clearly believe that both calculus and vectors are desirable preparation for their students. Representatives of physics programs spoke of the need to include geometric reasoning and spatial relationships or transformations in the curriculum. The difficulty of finding people with good skills in 3D geometry was referred to in several interviews we held, with particular reference to molecular modeling, computational chemistry, geology and materials science.

Across Canada, vectors is not offered at the high school level outside Ontario, though it is available in Quebec CEGEPs. The task force believes that teaching vectors in high school is an innovation Ontario can sustain through the new *Calculus and Vectors* course.

Another reason we felt a full course in calculus was not the best choice is that calculus – like vectors – is in fact taught to first-year university students. The study of calculus in high school creates a longer period (Grade 12 plus first year of university) to absorb key ideas and first exposes students to these ideas in the setting of a small classroom. That is where the value of high school calculus lies. There is more to be gained from exposing students to the concepts of both calculus and vectors in high school than from having the high school course attempt to replicate university-level calculus more fully.

The Content of the New *Advanced Functions* Course

The new Grade 12 *Advanced Functions* course would resemble the course of that name originally proposed for implementation in September 2006, except that the topic “rates of change” would now form part of the *Calculus and Vectors* course. The task force believes “rates of change” as a unit should be taught alongside the complementary calculus techniques. However, limited material on rates of change should be kept in *Advanced Functions* as part of the introduction of specific functions. *Advanced Functions* would also include trigonometry, building on what students have learned in Grade 11.

With the removal of calculus material from the course that is presently offered in Grade 12, the content of *Advanced Functions* will be less dense and students will have an opportunity to deepen their grasp of the remaining topics. As a result, *Advanced Functions* should provide sound preparation for students who elect *Calculus and Vectors* as a fifth course.

Substantially removing “rates of change” would allow *Advanced Functions* to include some material on either geometry or discrete mathematics. Some possibilities would be transformational geometry or conics and loci (which is being removed from the Grade 11 university preparation course in September 2006 and has often been taught as a topic for independent study). These options might benefit students interested in physics programs. Another possible topic area is elementary discrete mathematics, which is relevant to computer science programs.

If possible, consideration should be given to moving some geometric topics into the new Grade 11 *Functions* course, to provide better flow between the geometry in the Grade 10 academic course, *Principles of Mathematics*, and the proposed Grade 12 vectors content. Care must be taken not to reintroduce the problem of an over-packed curriculum, and some interchange of topics between Grades 11 and 12 may be advisable. Ideally, this rebalancing would be implemented in September 2007, but if that proves impossible then it would be important to revisit this in the next round of curriculum revision.

Recommendation:

5. That further consultations take place about what material should replace “rates of change” in *Advanced Functions*, and that consideration be given to moving topics between Grade 11 university preparation *Functions* and Grade 12 *Advanced Functions* in order to better balance both courses.

Improving the *Mathematics of Data Management* Course

Calculus is one of humanity's great intellectual achievements. It is also a necessity to advanced study in such fields as physics, engineering and finance. However, students going on to university in psychology, biology, commerce or many other areas may be better served by a statistics course than by one in calculus.

The current *Mathematics of Data Management* course engages students who might otherwise not take a Grade 12 mathematics course, and its statistical content has relevance that is clear to students. We talked with employers who were looking for, but generally not finding, entry-level employees with a statistical background. While such a course at the high school level is not offered in other jurisdictions, the task force feels that it is a useful and sustainable innovation for Ontario.

Recommendation:

6. That the course *Mathematics of Data Management* be retained in essentially its current format.

But while we recommend continuing this course, we also heard of several areas for improvement if this course is to reach its potential. Among the suggestions are:

- The content of the course should be expanded to include linear regression, providing an opportunity to revisit the topic of linear or quadratic functional relationships.
- Support from the ministry is needed around the ethical issues that arise in the collection of survey data by students.
- Better data is needed for projects. The course emphasizes techniques that apply to numerical data, while categorical data is usually all that is available for projects.
- “Science fair” type experiments could be given more prominence, both as a source of numerical data and as an exercise in data collection.

We feel that addressing these issues is better done through an ongoing process of consultation and evolution, rather than a periodic five-year cycle of curriculum revision.

Recommendation:

7. That the ministry appoint an ongoing statistical advisory group, with representation from teachers and from statisticians (both academic and practising), to bring forward recommendations for improving the delivery of *Mathematics of Data Management*.

This advisory group would perform its role over an extended period of time, leading to both incremental change to the *Mathematics of Data Management* course and improvements in the support and resources for teachers of this course. We understand that ministry staff have already held preliminary meetings with teachers and statisticians and we encourage the ministry to formalize this dialogue through a more structured process.

Mathematics of Data Management is likely to continue to attract a high percentage of students who do not go on to study mathematically intensive topics in university. Any modifications to the course or its delivery should be made with that in mind.

College Preparation and Workplace Stream Courses

The feedback we received on the proposed changes to the college preparation and workplace stream courses was broadly favorable. These alternative pathways prepare students for college programs in the skilled trades or technology, for example, or for direct entry into the workforce in fields ranging from retail clerk to automotive assembly.

In our consultations, it was suggested that the current Grade 11 *Mathematics of Personal Finance* college preparation course is a nice self-contained course for students taking no mathematics beyond Grade 11. But for students going on to take Grade 12 mathematics, we were told that the revised courses are better balanced, provide more continuity and offer better preparation for college study.

Though many of our conversations dealt with preparation for mathematically intensive postsecondary studies and related careers, we found a strong interest in mathematics among a wide range of employers in many different fields. The task force heard, for example, about the demand for employees who are comfortable doing mathematical calculations and using mathematical formulas. We heard about the importance of

accuracy and precision in carrying out computations and the role of estimation in spotting errors. We were told that an understanding of statistics is invaluable in maintaining quality control. It was also observed that high school mathematics is sometimes forgotten by the time students reach the workplace. Linking mathematical techniques to concrete examples of how these techniques could be applied on the job might improve retention.

The task force is satisfied that the planned revisions to the college and workplace mathematics courses in Grade 12 will go a long way towards meeting these diverse needs.

We consulted within the college system, though not as extensively as we would have wished because of the strike underway at the time. The major concern for colleges is not the proposed curriculum, but the fact that Grade 12 *Mathematics for College Technology* has such poor enrolment. This is a course college technology programs very much want their students to take, but low enrolment prevents them from requiring it. The Grade 11 curriculum changes being implemented in September 2006 will create a pathway from the Grade 10 applied course in mathematics to Grade 12 *Mathematics for College Technology*. Colleges, however, are not convinced that this alone will solve the problem. We will return to this issue in the next section of the report.

Recommendation:

8. That the Grade 12 courses *Mathematics for Work and Everyday Life*, *Foundations for College Mathematics* and *Mathematics for College Technology* be implemented essentially as currently planned.

IMPLEMENTING OUR RECOMMENDATIONS

Turning the recommendations of the task force into action will involve significant adjustments in two complex systems – the high school system and the postsecondary education system. Needless to say, change of this scope will pose challenges.

The student is ultimately the focal point of both systems. A key priority will be to ensure that implementation truly serves the needs of all students and treats them fairly.

Time Is of the Essence

Time is short. Students who enter universities and colleges in September 2008 will start Grade 12 in September 2007 and will select their high school courses in February or March 2007. While universities won't release official information to the Ontario Universities' Application Centre until May 2007, they will make decisions about course offerings and entrance requirements and prepare print advertising materials much earlier, most likely by January 2007. Universities will require at least two months to decide on entrance requirements, plus another month if coordination is needed between universities. For example, engineering faculties may work together to make entrance requirements at least partially uniform across Ontario. We heard that many parents already do not fully understand the mathematics pathways leading to various postsecondary options. If the release of entrance requirements is delayed, it will only make this problem worse.

The task force recommends that university and college deliberations about entrance requirements and changes in course content begin in September 2006 and proceed expeditiously. This schedule will require institutions to set up the appropriate committees before that date. These committees should look at all courses closely tied to the high school curriculum, including but not limited to first-year courses in mathematics, statistics, economics, physics and engineering.

These timelines will also require the ministry to publicly release information about the planned Grade 12 mathematics courses by early September 2006 at the latest. The ministry information could take the form of a broad course outline. University and college decisions about admission requirements could then take place in parallel with the ministry's confidential feedback process during the writing of the detailed curriculum.

Recommendation:

9. That the ministry release information about revised Grade 12 mathematics courses in two parallel processes, no later than September 5, 2006. One process would be the normal confidential review of the detailed curriculum proposals. The other would be a public release of short course summaries. On the basis of these summaries, universities and colleges could start to evaluate their entrance requirements and prerequisite structure.

Recommendation:

10. That academic vice-presidents and deans of postsecondary institutions ensure that committees are formed prior to September 2006, both to review entrance requirements and prerequisites and to determine how courses should be changed in response to the revised high school curriculum.

Recommendation:

11. That academic vice-presidents and deans consider the possibility of coordinated decisions about entrance requirements for specific programs (e.g., engineering) and plan timelines in such a way as to allow consultations on common requirements between universities.

In support of the latter two recommendations, academic vice-presidents or deans may wish to consider inviting members of this task force or ministry officials to attend their periodic meetings in spring or summer 2006.

University Decisions on Admission Requirements

Once the universities know what the new curriculum contains, they will make decisions about entrance requirements and course prerequisites. Specifically, they may make *Calculus and Vectors* a requirement for admission, or they may not. Outside Ontario and Quebec, many university programs recommend calculus for admission but very few actually require it (engineering in Alberta is a program that does). Based on this experience, we anticipate that many Ontario programs that currently require *Advanced Functions and Introductory Calculus* will move to require *Advanced Functions* and leave *Calculus and Vectors* as a “strongly recommended” course.

One concern is that students who have not taken *Calculus and Vectors* may find themselves at a disadvantage in a classroom with students who have.

Recommendation:

12. That mathematically intensive programs where some but not all entering students will have taken *Calculus and Vectors*, universities should consider how to provide additional courses or additional support for students who have not taken this course.

An example of this kind of support is the addition of an extra hour per week to the university calculus course taken by these students. The University of British Columbia and the University of Toronto have such programs now.

Another example is found in many science courses in western Canada, where university teachers anticipate mathematical techniques that will be explored later in mathematics courses. If done with care, through a concise explanation and an indication of when the technique will be studied in depth, students can readily follow such a presentation.

Course Availability and Equity Issues

We expect that university programs currently requiring *Geometry and Discrete Mathematics* will require *Calculus and Vectors*, but what about programs that once required *Geometry and Discrete Mathematics* but have recently dropped it? We expect they will require *Advanced Functions*, but will have concerns that *Calculus and Vectors* will not attract enough students (as happened with *Geometry and Discrete Mathematics*). And if postsecondary programs do not require *Calculus and Vectors*, students may feel it is not as valuable a course, or that their grades will suffer if they choose it over *Mathematics of Data Management*. Enrolments may then suffer, threatening the viability of the new course in smaller schools. If this transitional issue is not addressed, then the new course will get off to a bad start and it could be difficult to recover.

For two reasons, we feel that *Calculus and Vectors* stands a much better chance of success than *Geometry and Discrete Mathematics* did. First, calculus has proven sustainable in other provinces (though it is true that they do not offer a course such as *Mathematics of Data Management* as an alternative). Second, calculus is not seen as disconnected from the content of science and engineering programs, as *Geometry and Discrete Mathematics* is. Students and parents perceive calculus as highly aligned with the skills necessary for success in those programs.

However, the new calculus course probably will not be taken by as broad a range of students as has historically been the case in Ontario. We expect *Calculus and Vectors* to appeal mainly to students targeting mathematics-intensive programs such as engineering, science, finance and of course mathematics itself. We anticipate that the *Advanced*

Functions course will attract enrolment comparable to that of *Advanced Functions and Introductory Calculus*, while enrolment in *Mathematics of Data Management* will drop somewhat and *Calculus and Vectors* will record higher enrolment than *Geometry and Discrete Mathematics*.

Still, it is possible that enrolments will not be high enough to allow all schools to offer *Calculus and Vectors* every year. If this happens, questions of accessibility and equity will arise. These questions will be especially urgent if *Calculus and Vectors* is required for entry to university programs. Both universities and the ministry must address these issues.

Where *Calculus and Vectors* is an entrance requirement, we believe universities should nevertheless accept students who did not have access to this course and should find ways to help them close the gap. An example of a university solution is found in engineering at the University of Alberta, which requires calculus for admission. Students without high school calculus have the option of entering a parallel science program and then transferring into engineering in second year, once they have succeeded at university-level calculus. For some students this means taking summer courses or an additional year of study, but for many it imposes no additional course burden.

Recommendation:

13. That if *Calculus and Vectors* is an entrance requirement for a university program, the university involved should put in place mechanisms to ensure that students are not denied entry simply because the required course was not available to them.

On the ministry side, special steps should be taken to ensure that students in small schools or rural boards have access to *Calculus and Vectors*. E-learning or sharing courses between schools may help in some cases. However, this will not be appropriate for all students, and such measures are currently not widespread enough to address this problem reliably. When these alternatives are not viable, the task force believes schools should be allowed to offer *Calculus and Vectors* even to small groups of students.

A comparable challenge exists with *Mathematics for College Technology*. It is not feasible for high schools with relatively few college-bound students to offer this course, much as colleges value it. In these circumstances, the ministry should permit the course to be offered to smaller classes than usual.

Similar measures will likely be necessary throughout the French-language school system. French-language boards must put in place strategies to ensure that students have access to all courses generally required or recommended for admission to postsecondary programs. Where needed, boards should receive extra resources to do this, in recognition of the unique circumstances of the francophone community.

Recommendation:

14. That the ministry take steps to ensure that courses required or recommended for entry into programs at a substantial number of universities or colleges are available to all qualified Ontario students. In particular, financial support should be provided to schools where demand is low to allow them to offer such courses to smaller groups of students than normally permitted. This is especially critical in the first year the revised curriculum is implemented.

Special Measures for the Transitional Year

In the first year of implementation, universities and high schools must both guess at course enrolment when setting entrance requirements and mounting courses, respectively. This is risky for both. Universities may be reluctant to require a fifth mathematics course if they fear doing so runs the risk of not attracting students. Schools may be reluctant to offer such a course out of concern that universities won't require it, so students won't enrol. Neither scenario is in the best interests of Ontario students.

The true demand for the new university preparation mathematics courses will only become clear after the first, transitional year – 2007-08. Whether or not the above recommendation about smaller class sizes is adopted for the longer term, having it in place during the first year would ensure that Ontario students truly have choices and that the new courses have a fair chance to establish themselves.

The same issue arises in the college system, with the *Mathematics for College Technology* course. Colleges feel their students need this course, but low student interest has kept many high schools from offering it. But until a school has offered this course on a regular basis, it will be hard to attract students. Even if ministry support for small class sizes is temporary, it might be of great help in getting this course firmly established. At that point college technology programs might start requiring it for admission, reinforcing the demand.

What about Textbooks?

When implementing new courses, the availability of textbooks is always a challenge. In the first year, we believe that a combination of the current *Advanced Functions and Introductory Calculus* and *Geometry and Discrete Mathematics* textbooks can be made to serve. By the second year, new textbooks should be developed.

In the first year, teachers will be looking for guidance on how to use the existing textbooks and resources. An example of the kind of resource that could help is the guide to planning the Grade 11 university/college mathematics program for 2006-07. It identifies where expectations were taught in the previous curriculum so that teachers know where to look for ideas as they organize the new course.

Perceptions of University Attitudes

Rightly or wrongly, a widespread perception exists among high school teachers that universities pay little attention to the high school curriculum and will teach what they've always taught without adapting to changes in the preparation of their students. Many university students visit their high schools and tell their teachers about their experiences. Teachers report that many students feel the universities did not adapt courses to the changes that followed the elimination of the OAC curriculum. There is much anxiety among teachers that the same will be true following the pending revisions to the Grade 12 courses.

The most recent examples of this phenomenon are reports about university instructors who assume students know how to differentiate trigonometric functions. This topic left the

curriculum in 2003 with the end of the OACs. A longstanding example is integral calculus, which has not been in the high school curriculum (except as an optional topic) since 1985. Students report some universities where familiarity with integration is assumed. Yet even under the OAC curriculum, many schools did not teach it, or covered it only briefly, since it was not part of the core curriculum. University programs that behaved as though all students knew this material did a disservice to many of their students.

A consequence of the perception that universities do not know what is in the high school curriculum is that teachers deviate from the curriculum in order to teach what they think universities expect students to know. This tends to overload course content even more than the curriculum itself already does, at the expense of time spent acquiring deep understanding, problem-solving ability or even basic algebraic skills. Since additional topics are added in an inconsistent way, this tendency also contributes to the variation universities encounter in the knowledge and preparation of their students. That in turn complicates the delivery of first-year university courses.

We emphasize that many programs at many universities do in fact carefully examine the high school curriculum. This is one reason integral calculus at the university level is often taught at a more deliberate pace than differential calculus. But the perceptions of students and teachers are strongly influenced by the programs and professors who do not show the same attention to the preparedness of their students.

We believe universities would be better served by setting clear and uniform expectations, so that what students are expected to know is understood reliably by students and teachers, at more than a superficial level. But to achieve this, universities must not only be aware of the high school curriculum, but must also be seen to be aware of it.

Recommendation:

15. That academic vice-presidents and deans make clear and public commitments to ensuring that the content of university courses is appropriate and accessible to students possessing the knowledge and skills laid out in the high school curriculum.

LONGER-TERM ISSUES IN MATHEMATICS EDUCATION

Our recommendations thus far have focused on what can be done in the short term to make the high school mathematics curriculum work better for students, postsecondary institutions and society as whole. Now we turn to changes that will take longer but will also contribute to this objective.

In the 2003 Programme for International Student Assessment (PISA) study, Ontario Grade 9 students scored lower in mathematics than their counterparts in Alberta, British Columbia and Quebec. PISA assesses the reading, mathematics and scientific literacy of 15-year-olds. Moreover, as discussed earlier, strong evidence suggests that the performance of Ontario high school graduates in university mathematics has declined since the abolition of the OACs.

These outcomes are not acceptable, but reversing them will take time. Short-term changes will help, but they are not the whole answer. The task force believes longer-term measures such as a comprehensive review of the K-12 mathematics curriculum and improvements in teacher education are also essential.

Comprehensive Curriculum Review

The curriculum is the blueprint for teaching and learning in the classroom. We feel strongly that Ontario's mathematics curriculum must be examined as a whole and reviewed in its entirety.

In our conversations, many stakeholders observed that the options for revision to the Grade 12 curriculum were limited because changes to earlier grades had already been determined. We realize there were reasons for the staged approach. Grades 1-8 were dealt with more quickly because of concerns about the transition from elementary to secondary school. Originally, the reviews of Grades 9-12 were proceeding on the same track, but the ministry accelerated the process for Grades 9-10 to address high failure rates in the applied stream. Then the decision was made to delay implementation of Grade 12 changes to permit this task force to do its work and the ministry to write a curriculum in light of our findings.

While we understand why the revisions evolved this way, the effect has been to fragment the review process. At the very least, once the currently envisioned changes are made, there should be an opportunity to sit back, take stock of where the process has led, and openly consider where improvements can still be made across all grade levels.

In recent years there has been no opportunity for systematic discussion of the entire K-12 mathematics curriculum. For example, there has been no vehicle for an in-depth comparison of Ontario's elementary and secondary curriculum with Quebec's. A number of observers told us that the Quebec mathematics curriculum is roughly a year ahead of Ontario's by the time students reach Grade 9. In addition, as noted earlier, first-year students from Quebec are now apparently outperforming their Ontario counterparts in some universities.

At the very least, the full Grade 7-12 curriculum should be reviewed as a unit. This is a longer-term undertaking than determining what the Grade 12 curriculum will be in September 2007. But the task force believes it will be worth the time and effort. This process should start now, rather than waiting for the next round of a five-year cycle.

Recommendation:

16. That a thorough evaluation of the entire Grade 7 to Grade 12 mathematics curriculum as a whole be carried out.

The task force heard much criticism of the haste with which the present high school curriculum, phased in starting in 1999-2000, was created. Many topics were dropped in compressing the high school curriculum from five to four years. For example, geometry is all but absent from the current curriculum, and little space is left for discrete mathematics despite its increased prominence in the economy. (Indeed, our recommendation to replace *Geometry and Discrete Mathematics* with *Calculus and Vectors* is another step in this direction.) Only a minimum of attention is devoted to algebraic skills. In the academic and university preparation pathway, little is said about learning to generalize and to prove.

Surely a careful and thorough examination of the whole Grade 7-12 curriculum could make significant improvements, and raise the quality of our curriculum in comparison with that of other jurisdictions. Such a methodical review over several years could be expected to address inconsistencies in the curriculum from grade to grade. This process should also assess the achievements and drawbacks of other curriculums, including the effect on dropout rates, to determine what works and what doesn't.

The task force heard repeated calls for broader involvement in the curriculum review process. It is widely felt that the most recent round of mathematics curriculum review that began in September 2003 offered little if any scope for public engagement.

We have great respect for the individuals who carried out the most recent review, and we recognize that many valuable curricular improvements emerged from that process. Indeed, we learned after drafting our report that several of our recommendations were very close to changes the curriculum-writing team had been preparing to implement as a result of the feedback received during the regular review process.

Nevertheless, that process was not entirely transparent to those outside the ministry. The changes that the ministry team proposed for implementation in September 2006 were more substantial than had probably been contemplated at the start of the review process. In the view of the task force, changes of this magnitude require a longer period for consultation and feedback. In particular, issuing "white papers" describing the rationale for and direction of change would have made the review more transparent and open, and encouraged wider participation. Stakeholders could have been invited to comment on the general direction for reform, and later given the opportunity to comment on the detailed curriculum proposals.

The public release of the task force's report will undoubtedly generate feedback that will inform the next round of curriculum writing. In that sense, we see this report as contributing to the type of open review we recommend.

Recommendation:

17. That the next review of the mathematics curriculum be done at a slower pace and in a more open manner, including the publication of "white papers" to ensure comprehensive and considered feedback and debate.

It was reported to us that across Ontario, 65 per cent of aboriginal students drop out of high school. This is obviously a complex problem, but one factor appears to be that many aboriginal students fail to see the relevance of mathematics to their daily lives. The aboriginal perspective should be considered as part of the mathematics curriculum review to ensure that the curriculum is culturally sensitive. Teachers need to develop special strategies to help bridge the gap between aboriginal students' culture and key mathematical concepts, and learning resources should be provided to do this as well.

Recommendation:

18. That the ministry provide support for the implementation of the Grades 7-12 mathematics curriculum in aboriginal communities.

A Mathematics Curriculum Council

To sustain this long-term process to refine the curriculum, we should lay the foundation now. We need to put in place a comprehensive structure that will represent all stakeholders and enable them to work together. To this end, the task force proposes that a mathematics curriculum council be established.

One priority for the council will be to monitor and share information about the postsecondary reaction to the revised curriculum – such as which courses are required or recommended, and the support measures implemented. We will also need data on the progress of students entering university or college in September 2006 (under different entrance requirements than before), the success of students in the new Grade 12 courses starting in September 2007, and the success of those students when they enter university or college in September 2008.

Recommendation:

19. That the ministry establish a mathematics curriculum council prior to September 2006 that would include representation from teachers, students, school boards, parents, industry sectors and relevant university and college disciplines.

The postsecondary membership would reflect fields that make extensive use of senior high school mathematics courses – such as engineering, computer science, finance, mathematics, biology, physics, chemistry, economics, business, technology and skilled trades.

Some high school, university and college mathematics teachers now interact regularly. Examples include the Grand Valley Association in the Waterloo area and the Fields Institute mathematics education forum in Toronto. Nevertheless, for most teachers the only opportunity to discuss the mathematics curriculum with university or college faculty arises in connection with the ministry's five-year curriculum review. One such conversation every five years is not enough. Many topics would benefit from discussion, among them the disparity in the use of technology between high school and university.

Recommendation:

20. That the mathematics curriculum council sponsor at least one open meeting each year, where teachers and representatives of postsecondary institutions would have an opportunity to discuss issues arising out of the high school curriculum.

One option is to invite the Ontario Association for Mathematics Education to host such a public session at its annual meeting.

We now turn to further proposals for change over the longer term.

Taking More Grade 11 Mathematics Courses

We heard a suggestion to encourage students to take two mathematics courses in Grade 11, followed by two or more mathematics courses in Grade 12. We were unconvinced that

students would choose this option in numbers large enough to justify basing the curriculum on it. Moreover, this emphasis on mathematics would crowd out courses from other disciplines that many university programs (including engineering, science and business) also want their students to take. Nevertheless, we feel there are some students who would be interested in and would benefit from taking two mathematics courses in Grade 11.

For example, topics could be adjusted so Grade 11 mathematics could be taken concurrently with *Mathematics of Data Management* by students performing well in Grade 10. Or, in semestered schools, an *Advanced Functions* section could be offered in the second term following a section of Grade 11 mathematics. *Advanced Functions and Introductory Calculus* is currently available to Grade 11 students in some schools, and the option of offering the proposed *Advanced Functions* course in Grade 11 should be explored.

A word of caution, however. Schools should carefully consider the cognitive maturity of individual students before encouraging them to take either *Mathematics of Data Management* or *Advanced Functions* while in their third year of high school.

Recommendation:

21. That the ministry explore ways to allow students to more easily take two mathematics courses in Grade 11.

Improving the Education of Mathematics Teachers

Among the most promising ways to improve student preparation in mathematics is to enhance the training of mathematics teachers, both pre-service and in-service. This has often been said, but it bears repeating.

Shortages of teachers with advanced mathematics training persist in Ontario. Many elementary school teachers are fearful of mathematics, but still must teach it. More attention should be paid to the special demands that mathematics instruction places on teachers. And more encouragement should be given to teachers to develop mathematics

skills that will translate into classroom improvements. Teachers already in the system need more professional development, both in dealing with the content of specific courses and in developing and sharing rich tasks for in-depth learning.

Recommendation:

22. That greater attention be paid to the particular demands of teaching mathematics, through teacher recruitment, pre-service education and in-service education.

The task force believes the priorities for professional development at the senior high school level should be calculus, statistics and vectors. This will ensure that teachers are well equipped to teach the Grade 12 courses we propose. Emphasis should be placed on both subject expertise and pedagogical expertise – on both what is taught and how it is taught.

We received many comments that students were struggling with calculus and part of the reason may be the teacher's approach to the subject. Some teachers apparently teach calculus as a set of algorithms to be mastered, while failing to convey the underlying and unifying concepts. Professional development is imperative where this is the case.

Statistics is also a difficult subject for students and teachers alike. It is one that many mathematics teachers may not have studied themselves. We also heard that many teachers who are comfortable teaching calculus may not be comfortable with vectors. They may not have taught vectors before or they may have studied vectors in less depth, if at all, in university. With statistics and vectors, both content knowledge and teaching styles will require attention. And since statistics courses are heavily project-oriented, teachers who are accustomed to test-based evaluation will need to become familiar with project-based assessment.

Though enhanced teacher training is central to improving the mathematics understanding of Ontario students, it should be underlined that this is a long-term objective. The curriculum to be introduced in Grade 12 in September 2007 will be delivered by largely the same teachers with the same skills as we have today.

IN CLOSING

The task force would like to thank the dozens of individuals from many walks of life who generously agreed to be interviewed as part of our consultations. It is obvious that they care as deeply as we do about what students are learning in our classrooms. We simply could not have written this report without them. We also wish to thank the ministry staff who supported the task force and gave unstintingly of their time in the pursuit of an improved mathematics curriculum.

Ontario deserves the best and most relevant mathematics curriculum possible, one that fosters success for all students and also meets the needs of future leaders in the knowledge-based industries that are driving economic growth. We are confident our recommendations to the minister will help achieve this vision and secure Ontario's future.



Tom Salisbury, Chair



Beverly Farahani, Member



André Ladouceur, Member



Manon Lemonde, Member



Husein Panju, Member

Appendix I

1. Data on completions in university preparation mathematics courses in Ontario from 1997-98 to 2004-05.

The tables below provide data on course completions and success rates in Ontario university preparation mathematics courses from 1997-98 to 2004-05. While direct comparisons between the previous 5-year secondary program with Ontario Academic Credits (OACs) and the current 4-year secondary program are difficult to make, the tables indicate general trends in course completions. Note: the numbers for 2002-03 are significantly higher due to the double cohort year, when the last students graduated from the 5-year program along with the first graduates from the 4-year program.

Between 1997-98 and 2004-05 there was a significant increase in the number of successful completions in Calculus/Advanced Functions and Introductory Calculus, a moderate increase in Finite Mathematics/Mathematics of Data Management and a significant decrease in Algebra and Geometry/Geometry and Discrete Mathematics. Over the 8-year period student success rates were slightly lower in Calculus/Advanced Functions and Introductory Calculus than in Finite Mathematics/Mathematics of Data Management and Algebra and Geometry/Geometry and Discrete Mathematics

Table 1. Calculus (MCAOA)/Advanced Functions and Introductory Calculus (MCB4U)

Year	Course/courses	No. of Successful Completions (Marks \geq 50%)	No. of Failures (Marks $<$ 50%)	Total: Successful Completions + Failures	Success rate
1997-98	MCAOA	31,403	4,708	36,111	87%
1998-99	MCAOA	31,439	2,964	34,403	91%
1999-00	MCAOA	29,812	5,378	35,190	85%
2000-01	MCAOA	30,694	5,294	35,988	85%
2001-02	MCAOA/MCB4U	32,971	4,343	37,314	88%
2002-03	MCAOA/MCB4U	58,243	7,650	65,893	88%
2003-04	MCB4U	40,220	5,935	46,155	87%
2004-05	MCB4U	40,956	5,266	46,222	89%

Table 2. Finite Mathematics(MFNOA)/Mathematics of Data Management(MDM4U)

Year	Course/Courses	No. of Successful Completions (Marks \geq 50%)	No. of Failures (Marks $<$ 50%)	Total: Successful Completions + Failures	Success rate
1997-98	MFNOA	24,705	2,786	27,491	90%
1998-99	MFNOA	24,552	1,870	26,422	93%
1999-00	MFNOA	22,370	3,098	25,468	88%
2000-01	MFNOA	23,655	2,882	26,537	89%
2001-02	MFNOA/MDM4U	23,473	2,539	26,012	90%
2002-03	MFNOA/MDM4U	34,125	3,052	37,177	92%
2003-04	MDM4U	27,455	2,395	29,850	92%
2004-05	MDM4U	28,072	2,085	30,157	93%

Table 3. Algebra & Geometry (MAG0A)/Geometry & Discrete Mathematics (MGA4U)

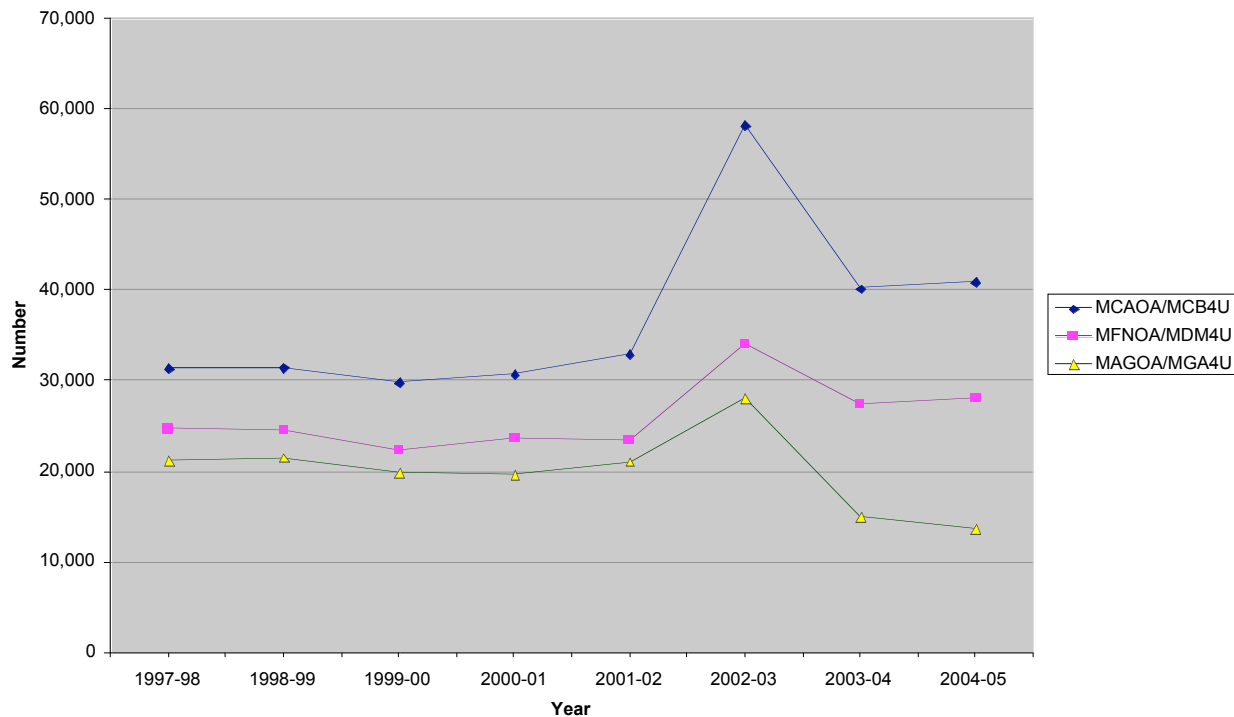
Year	Course/Courses	No. of Successful Completions (Marks >= 50%)	No. of Failures (Marks < 50%)	Total: Successful Completions + Failures	Success rate
1997-98	MAG0A	21,220	2,228	23,448	90%
1998-99	MAG0A	21,523	1,600	23,123	93%
1999-00	MAG0A	19,826	2,699	22,525	88%
2000-01	MAG0A	19,678	2,490	22,168	89%
2001-02	MAG0A/MGA4U	21,126	2,063	23,189	91%
2002-03	MAG0A/MGA4U	28,087	2,484	30,571	92%
2003-04	MGA4U	14,981	1,186	16,167	93%
2004-05	MGA4U	13,759	1,020	14,779	93%

Source: Information Management Branch
Data includes Public and Roman Catholic Schools

The graph below shows the trend in successful completions in Ontario university entrance mathematics courses from 1997-98 to 2004-05.

Figure 1.

**Successful course completions in OAC and Grade 12 University mathematics
1997-98 to 2004-05**



Source: Information Management Branch, Ministry of Education

Legend	
MCAOA/MCB4U	Calculus/Advanced Functions and Introductory Calculus
MFNOA/MDM4U	Finite Mathematics/Mathematics of Data Management
MGAOA/MGA4U	Algebra & Geometry/Geometry & Discrete Mathematics

2. International Student Assessment Results: Programme for International Student Assessment (PISA)

The Programme for International Student Assessment (PISA) is sponsored by the Organisation for Economic Co-operation and Development (OECD). PISA assesses the reading, mathematics, and scientific literacy of 15-year-old students. In the latest PISA study, 15-year-old students in Ontario scored lower in mathematics than their counterparts in three other provinces. See Table 4 below.

Table 4. Average PISA scores on Combined Mathematics, selected provinces.

PISA 2003	
Selected Canadian provinces	Average score on Combined Mathematics scale
Alberta*	549
British Columbia	538
Quebec	537
<i>Canadian average</i>	532
Ontario	530

Source: Extracted from Table B1.1 Canadian Results of the OECD PISA Study 2003

*Note: Alberta's score is statistically higher than the other scores listed.

3. Data on acceptances to Ontario universities by university preparation mathematics courses, 1997-2005

Secondary students confirm acceptance to one institution in response to offers of admission to universities in Ontario. The total of confirmed acceptances by students applying from secondary schools to Ontario universities increased from 41,493 in 1997 to 57,173 in 2005. This increase is partly explained by the fact that two cohorts of secondary school students completed for admission to Ontario universities in 2003-04. In addition, total university enrolment is increasing because of the double cohort flow-through, an increase in the 18-24 year-old population and rising participation. The cohort of secondary school applicants to university in 2006 is 27.2% higher than the cohort of students applying to university from the old secondary school curriculum in 2001.

Patterns of course-taking in mathematics have changed over time, in response a number of factors, such as a compressed curriculum resulting from changing from a 5-year to a 4-year secondary school program and changes in university entrance requirements.

Data on confirmed acceptances is found in Table 5 below for students who applied directly to Ontario universities from public and private secondary schools in Ontario.

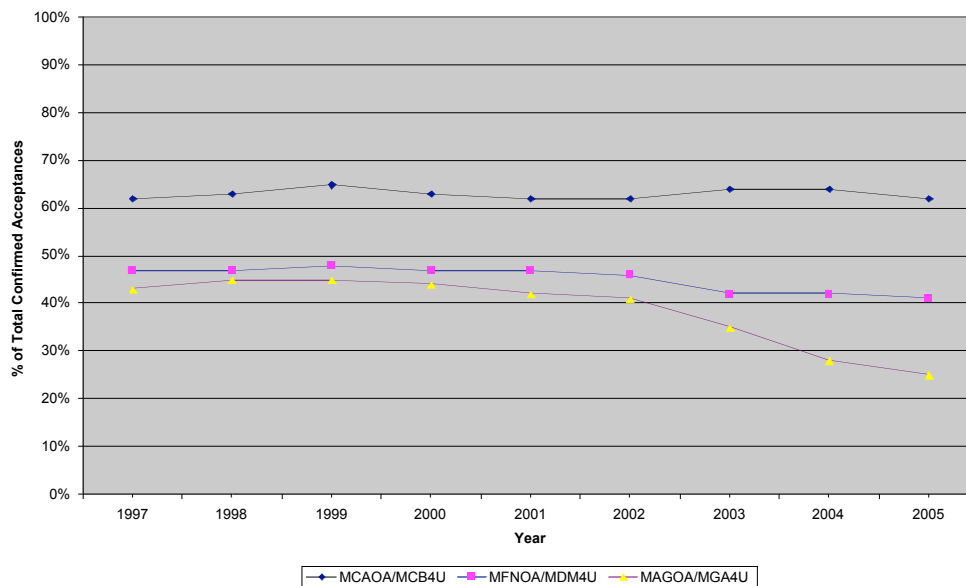
Table 5. Confirmed acceptances to Ontario universities by university preparation courses in mathematics, 1997-2005

Year	Total Confirmations	MCAOA/ MCB4U No. of Confirmations	MCAOA/ MCB4U % of Total Confirmations	MFNOA/ MDM4U No. of Confirmations	MFNOA/ MDM4U % of Total Confirmations	MAGOA/ MGA4U No. of Confirmations	MAGOA/ MGA4U % of Total Confirmations
1997	41,493	25,540	62%	19,532	47%	17,989	43%
1998	41,999	26,467	63%	19,845	47%	18,778	45%
1999	45,611	29,422	65%	21,796	48%	20,608	45%
2000	45,151	28,357	63%	21,322	47%	19,922	44%
2001	46,135	28,461	62%	21,619	47%	19,421	42%
2002	53,621	33,310	62%	24,876	46%	21,812	41%
2003	76,093	48,418	64%	32,159	42%	26,524	35%
2004	56,603	36,003	64%	23,810	42%	15,749	28%
2005	57,173	35,548	62%	23,616	41%	14,179	25%

The proportion of university-bound students who took Calculus/Advanced Functions and Introductory Calculus has kept pace with increases in postsecondary enrolment from 1997-2005. While more students took Finite Mathematics/Mathematics for Data Management over this time period, the proportion of students dropped from 47% to 41% of the total. The number of students who took Algebra and Geometry/Geometry and Discrete Mathematics decreased and the proportion of students with this course overall dropped from 43% to 25%. Universities are most likely to confirm acceptance to students who have a Calculus course. The percentage of confirmed acceptances by students with credits in university preparation mathematics courses from 1997 to 2005 is shown in the graph below.

Figure 2.

Confirmed acceptances with pre-university math as a percentage of total confirmed acceptances to Ontario universities, 1997-2005



Source: Ontario Universities Application Centre