# Probabilistic Investing: or How to Win the Globe and Mail's Stock Picking Contest (50% of the time)<sup>1</sup>

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# **1. Introduction and Motivation:**

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For the past eight years the *Globe and Mail* newspaper has held an annual stock picking contest entitled My One and Only. In this competition, which starts on January 1<sup>st</sup> of each year, a variety of financial commentators, money managers and even academics are asked to select one stock – from the universe of stocks trading above \$1 – of any public company quoted on the Toronto Stock Exchange (TSE). In addition to human participants, a completely random selection is added to the competition as well. The performance of all entries are tracked daily on a popular website and reported on quarterly in the actual newspaper, but the winner of the contest is the sole individual with the best performing stock at the end of the year, based on the close of trading on December 31<sup>st</sup>. The final results of the contest are announced with much fanfare and publicity on the front page of the *Report on Business* section in the first week of the subsequent year. Aside from the publicity (negative or positive) from being part of the game, the winner's only reward is a coffee mug, compliments of the Globe and Mail.

<sup>&</sup>lt;sup>1</sup> This article is being prepared for submission to the *Canadian Investment Review*. Please do not quote or reference without author's permission. Revisions to this paper can be downloaded from www.ifid.ca.

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In 2002 and then again in 2003, a Professor of Finance at one of Canada's leading business schools (and one of the authors of this paper) won the contest by beating all other participants, as well as the TSE market index by a wide margin. And, while it is easy to dismiss such results as completely attributable to luck, there is in fact a well-developed theory behind optimal behavior in such a contest. A rational and cognizant player can substantially increase the odds of winning the investment contest by playing the game optimally. We will review this theory in detail and make the interesting argument that picking stocks to win an investment game is quite different from selecting securities for personal investment portfolio.

In other words, motivated by this investment contest and the surrounding public interest, our paper takes the opportunity to review the theory of stochastic games and provide some anecdotal evidence as well as rigorous insights into the best way to win the Globe and Mail's stock picking contest. Our main intellectual objective, however, is to illustrate the critical difference between a rational and prudent strategy for building wealth versus the optimal strategy for picking stocks in these all-or-nothing contests. And, while there are many such investment games in existence -- the oldest and most popular being the *Wall Street Journal's* quarterly analyst vs. dartboard contest – the national stature and exposure of the G&M contest and the involvement of one of the authors makes this an ideal case study.

The actual paper is organized as follows. Section 2 provides a gentle introduction to the theory behind investment contests, and demonstrates that the strategy with the best odds of winning is somewhat counter-intuitive. Namely, we demonstrate that one should pick securities with very high idiosyncratic (i.e. diversifiable) risk, but with zero or possibly negative beta; in the lingo of modern portfolio theory. This tactic is perfectly justified despite over 40 years of dogma which preaches no equilibrium compensation for taking on idiosyncratic risk in capital markets. Furthermore, we show that the *a priori* probability of winning such a contest under our suggested optimal strategy comes close to, but can never exceed, 50%. An unobtrusive technical appendix provides the necessary probability theory. Section 3 of the paper reviews the actual stock selections and choices of the

contest participants over the 8 years of the contest. By examining betas and volatilities, we show that most of the winning stocks -- or the stocks that had the highest probability of winning *a priori* -- did in fact conform to the above selection criteria.

Aside from theoretical insights, the practical lessons from this paper's analysis is that picking stocks with the sole objective of winning a contest should not be viewed as a proxy for providing any guidance on building a well-diversified investment portfolio. Professional investment managers and individuals who manage their own portfolios must understand and acknowledge the inherent statistical illusions induced by such games. Indeed, one might go so far as to argue that the optimal strategies in both these endeavors are orthogonal to each other. *Ex post* one might be tempted to ascribe talent to winners, and especially repeat winners, when in fact the odds strongly favor such an outcome by chance when the game is played optimally. Anyone who attempts to replicate or mimic the behavior of the so-called contest winners with real money is likely to be disappointed. The message is crystal clear. Do not try this at home.

# **2. What Does Theory Tell Us About Winning Investment Contests?**

Modern investment theory – as originally developed by Harry Markowitz and Bill Sharpe $3$  – argues that all stocks can be classified by a limited set of parameters, namely their expected return and covariance structure. And, although a priori it would seem that the optimal strategy for winning a contest is to locate the stock with the highest (best) expected return, the truth is far from being that simple. In fact, given the difficulty in estimating expected returns over very-short periods of time, the secret to winning such a contest (or at least maximizing the odds) is to *completely ignore* expected returns and focus exclusively on the covariance of the stock in question.

Traditional mean-variance framework underlies most of today's professional money management and is the springboard for our analysis. The technical appendix provides a detailed derivation of the probability of winning as a function of the expected return,

 $3$  We refer the interested reader to the book by Elton and Gruber (1995), and especially chapter 7, for a detailed discussion of modern portfolio theory and specifically the mathematics underlying mean-variance analysis.

standard deviation and correlation structure. This analytic machinery can be used to demonstrate that for any reasonable expectation of investment returns – which we argue is almost impossible to estimate over short contest horizons – the probability of winning is most sensitive to the covariance structure of the security. The secret to winning – if there is any secret – is to pick a stock that is highly volatile and negatively correlated to the rest of the pack. This implicitly maximizes the odds of an extreme ranking. When done properly, the extreme ranking which is induced by this strategy will result in either winning the contest (50%) or coming dead last (50%). And since true money is never at play in these games, a 50/50 chance is the best one can hope for. Recall that in contrast to real life where the difference between  $3<sup>rd</sup>$  or  $4<sup>th</sup>$  or  $5<sup>th</sup>$  place represents real dollars and cents in one's retirement portfolio or quarterly evaluation, there is little difference for those placing anything other than  $1<sup>st</sup>$ . Though, for a real money manager, as opposed to a finance professor, it might be argued that some utility is derived from finishing in the top half of players, even if one doesn't finish first. And, while some might argue the harm caused by a last place or similar finish, these risk-averse contestants have the option of simply not participating. In the language of decision theory, the objective of the contest is to maximize a binary utility function which either takes on the value of one (winning) or zero (not winning). This is quite different from utility functions which balance risk – a.k.a shortfall, standard deviation or regret – and expected returns. Maximizing a smooth and differentiable utility function leads to a diversified portfolio of individual stocks, as originally shown by Markowitz. A binary objective function induces a strategy which maximizes the probability of beating a stochastic benchmark, independently of the magnitude of loss or disappointment.<sup>4</sup>

 To understand the counter-intuitive implications of this strategy and the impact of volatility and correlation on winning, we offer the following simplistic example. Assume we are asked to participate in a contest with only two other participants, each of whom has already selected their stock. We denote the standard deviation (or volatility) of our

<sup>&</sup>lt;sup>4</sup> We refer the interested reader to the recent papers by Brown, Harlow and Starks (1996), Chevalier and Ellison (1997) as well as Busse (2001) or Goetsmann, Ingersoll and Ross (2003) for a theoretical discussion and some empirical evidence on the impact of financial incentives on professional portfolio managers in a tournament-like environment. Indeed, it appears that similar game-theoretic behavior and strategies might be optimal in the money management business.

competitors' stocks by the symbols { $\sigma_1, \sigma_2$ } and their mutual correlations by the symbol  $\{\rho_{12}\}\$ . Recall that volatility is a proxy for the range of possible values for the end-of-year investment returns, while correlation measures the extent to which the two move together. Correlation values range from  $\{-1 \le \rho \le 1\}$ . Volatility can be as low as zero, for a risk-free product and can goes as high as infinity, in theory.

We are now asked to join the contest – knowing our competitors' two picks – and must select a stock that is characterized by a standard deviation { $\sigma_3$ } and a correlation pair { $\rho_{31}, \rho_{32}$ }. For the purpose of this example, we limit our search amongst the universe of correlation pairs that are equal. In other words, we assume that  $\rho_{32} = \rho_{31} = \rho$ . The same idea would apply under a more complicated search. Figure #1 is a graphical illustration of the triangular relationship between our *yet-to-be-made* stock pick and our two competitors'. If, instead of 2 competitors we had 7, then Figure #1 would have a pyramid-like structure (in 7 dimensions) displaying all the possible correlations. While this all might appear somewhat abstract and removed from the process of stock picking – after all, where is the discussion of price-to-earning ratios, dividends or growth rates? – we believe this framework truly captures the essence of the competition.



With this model in hand, Table #1 provides a number of important insights into the optimal strategy for winning the contest, which differs markedly with best practices for portfolio construction. If we select a stock that is completely uncorrelated ( $\rho = 0$ ) with our competitors' stock picks and all contestants have the same investment volatility ( $\sigma = 1$ ), the table illustrates that the probability that we win the contest is exactly 1/3. This should be intuitive. There are three  $(2+1)$  stocks in the contest, they are all identical and symmetric, and therefore all have an equal chance of winning. The same 1/n probability would apply for n symmetric securities as well. For those readers interested in a rigorous proof, we refer them to equation (eq.7) in the appendix which must be generalized to  $\mathbb{R}^n$ .



In contrast to the symmetric  $1/n$  result, when our competitors' correlations are zero  $(\rho_{12} = 0)$  and we select a stock that is twice as volatile as our competitors' ( $\sigma = 2$ ), the probability of winning the contest goes from 33.3% to 39.8%. When our standard deviation is increased by a factor of ten  $(\sigma = 10)$ , the probability of winning then jumps to 47.7% In fact, one can show that as the standard deviation goes infinity, the probability of winning converges to exactly 50%. This is true regardless of the number of stocks in the contest. For example, if there are 8 stocks in the contest, the neutral symmetric probability of winning is 1/8 which is 12.5%. Increasing the volatility of the stock will rapidly drive our probability of winning to 50%, leaving the other 50% chance to be shared amongst our 7 competitors. Note that we are focusing on total volatility, which is the sum of both systematic (non diversifiable) and non-systematic (diversifiable) risk.

If we decrease the correlation ( $\rho \ll 0$ ) between our pick and our competitors' picks, the probability of winning increases to the same 50% upper limit, but at a much higher rate. Indeed, when the correlation between our selection and the two other picks is set to minus 70% ( $\rho = -0.70$ ), the probability of winning increases to 37.5% (from 33.3%) when the volatilities are identical. Note that we have restricted the correlation parameters in Table #1 to the range  $\{-0.7, +0.7\}$  so as to maintain an appropriate covariance matrix that is positive semi-definite. We refer the interested reader to the appendix where this is explained in greater detail.

In the opposite volatility direction, if our stock pick is half as volatile as our two competitors' ( $\sigma$  = 0.5), the probability of winning the contest is greatly reduced. In fact, when our correlation to our competitors' is set to a positive 70% value, our probability of winning drops to a mere 9.7% compared to the equal chances under a completely symmetric contest. In this case, i.e. when we are 70% positively correlated to both our competitors', but half as volatile, the remaining  $100\%$  - 9.7% = 93.3% percent of winning is split evenly between the two opponents, for an equal 46.65% chance of winning. Thus, a poor choice of either correlation of volatility on our end not only harms our chances of winning, but obviously greatly enhances our opponents' positions. Clearly, there is a substantial amount of game theoretical implication when this contest is played by rational investors, each fully cognizant of their opponents' rationality. A full analysis of the relevant game theory is beyond the scope of this article, and we refer the interested and brave reader to a series of papers by Browne (1999, 2000) for more information about maximizing the probability of beating the return from a given portfolio of stocks within a game-theoretic context. Our point here is to emphasize the impact of volatility above all.



 Table #2 provides information similar to Table #1, except that we assume the correlation between our opponents' stock picks is *negative* 25%, albeit with equal volatility. Once again our probability of winning the contest depends on our stock's volatility { $\sigma_3$ } as well as the correlation with the other two stocks.

 The most pertinent observation is that the probability of winning when our opponents have selected anti-correlated stocks is reduced at all levels of volatility. For example, in contrast to Table #1, if we select zero correlation with our opponents'  $(\rho = \rho_{31} = \rho_{32})$ , and we have all picked stocks with equal volatility, our probability of winning is reduced from 33.3% in Table #1 to 31.2% in Table #2. This might not seem like much of a change, but as we increase the number of contestants, the relative gap will become larger and more noticeable. It is important to note that the relationship between winning and correlation does not appear to be universally monotonic, even though the probability of winning does increase with reduced correlation in both Table #1 and Table #2. At extreme levels of opponents' correlations, there might be a reduced benefit to selecting even more negatively correlated stocks.

One thing is true regardless of all the other parameters in the system, the greater our stock's volatility, the greater our chances of winning. Once again, the intuition for this fact is as follows. When selecting an extremely volatile stock there are two possible outcomes: either our stock earns a very high (extremely positive) return, or we earn a very low (i.e. extremely negative) return. The odds of either outcome are roughly 50/50. If we achieve a favorable return and our overall correlation with our opponents' is negative, they will likely experience a worse relative to the mean, and we win (50% chance). This is why both high volatility and negative correlation is important. The first factor places us in the required extremes while the second factor places our opponents on the other side.

#### **2.1 Unknown Opponents, the Connection to Beta and Diversifiable Risk**

 One of the issues that arises when implementing such a strategy in practice, is that we are unlikely to be told in advance the name (and hence the historical volatility and

correlation) of our opponent's stock. However, if we assume they will be selected randomly from the available market -- and we obviously have to compete against the market index as a whole -- then we can focus on two hypothetical opponents. The first is the market itself, and the second is a stock with some unknown *beta* relative to the market.

Recall that modern portfolio theory dictates that diversifiable (i.e. idiosyncratic) investment risk is not worth bearing since it is not compensated in economic equilibrium<sup>5</sup>. Under this theory, the only risk that matters is systematic, and the expected return from any security is linearly related to its beta. The greater the beta -- which is the covariance of the security's return with the general market portfolio, scaled by the variance of the security -- the greater is the expect return. Thus, investors who are interested in maximizing expected returns without any concern for risk would optimally select securities with the highest beta. The mathematical definition of beta is:

$$
\beta_i = \frac{Cov(X_i, X_m)}{Var(X_m)} = \frac{\rho_{im}\sigma_i}{\sigma_m}.
$$

From a mathematical point of view, the beta of the stock is a function of three ingredients, we discussed earlier in our analysis, the correlation with the market portfolio, the standard deviation of the stock in question and the standard deviation of the market portfolio.

Now, as we argued above, the optimal strategy in a contest with a zero-one payoff function is to locate a security with a negative correlation  $\{\rho_{i_m}\}\$  to our competitors' (i.e. the market), and with as high a volatility  $\{\sigma_i\}$  as possible. Thus, the optimal strategy is to pick very volatile and possibly *negative beta stocks*. This, once again, is in stark contrast to the objectives of any portfolio or wealth manager who is interested in maximizing expected returns subject to some reasonable constraints on risk.

The theory is unambiguous in recommending a very risky strategy to maximize the probability of winning the contest. Indeed, whether the equilibrium expected return from the stock is 5%, 15% or even 20%, the covariance matrix will play a far larger role in the outcome of this, or in any contest for that matter. We now move on to examine the

<sup>&</sup>lt;sup>5</sup> See the recent paper by Goyal and Santa-Clara (2003) for recent empirical evidence that perhaps some idiosyncratic risk might be worth bearing.

mechanics and actual choices made participants in the *Globe and Mail* contest during the past 8 years.

# **3. Which Stocks Did Contestants Actually Choose?**

As we explained in the introduction, the contest operates in the following manner. In late December of each year the investment editor of the *Report on Business* tabulates the performance of the previous year's contestants and asks the top 4 or 5 to participate in the coming year's event and select another stock. In addition, the editor picks 4 or 5 other contestants (at random) from a broad range of practitioners and academics in Canada. This assures a reasonable amount of turnover in the pool of contestants (unless one of them continues winning.) Occasionally contestants decide not to participate in the following year's contest and voluntarily withdraw, even though they earned a top berth in the previous year. Each contestant must select a stock amongst all securities traded on the Toronto Stock Exchange (TSE), valued at more than \$1. To add an element of humor, the editors select a young child from the *Globe and Mail's* extended family and ask them to randomly toss a toy at the stock quote pages. This becomes yet another contestant<sup>6</sup>. In the first year of the contest (1997) there were a total of 8 participants, the list was expanded to 9 for the period 1998 – 2003, and last year the total came to 10. Another complication that has only recently been added is the option to 'cash in' at the end of each quarter. And, while this option has never been exercised by any of the participants, it is quite conceivable that a large enough return in any given quarter might induce a participant to take the money off the table and cash-out. This further complicates the optimal strategy since one must now decide if and when to cash in, but this will not reverse our main conclusion about the optimality of high volatility and/or negative beta strategies.

Tables 3a and 3b display the selections made by all participants during the past 8 years, together with the standard deviation (volatility) and betas of the selected stock which we have computed using textbook methods. The stocks are listed in the order of

 $6$  In fact, in the year 2000 the winner of the contest was a windup toy Santa who enviously picked the company Denbury Resources and earned a total return of 179% during the year.

their final ranking and return. Not surprisingly, the winning stocks always outperformed the S&P/TSX market by a very wide margin, even in years when the market as a whole dropped in value. In most years the winning stock earned well in excess of 100% return.





Table 3b: Stocks Selected by Contest Participants, Their Volatility and Final Return.

**Notes to Table 3a, 3b:** The Betas and volatilities in this table are calculated based on 12 months of historical returns prior to the start of the contest. For example, for Gulf Canada Resources, which was selected for the 2001 year contest, the beta and volatility figures are calculated based on year 2000 returns. However, calculations for AXX and SBI (1999 Contest) are based on 1999 returns, and calculations for IAG (2001 Contest) are based on 2001 returns. These exceptions are due to the lack of a complete set of data for the prior years, likely because of demutualizations or IPO's. Additionally, where "N/A" is stated in the table, it is due to a lack of stock-price data.

The common and erroneous perception from a casual examination of these tables is that financial experts -- or at least the participants in these contests -- are able to select winning stocks that consistently beat the market and earn handsome returns. Indeed, quite a few of the winners have marketed their success in this contest as evidence of their stock picking prowess. Some winners have added this achievement to their public biography and CVs. Yet, as we have argued in the previous section, perceptions vis a vis optimal contest strategies are from the harsh reality of picking a well diversified portfolio of stocks that earns positive risk-adjusted returns in the long-run.

 In the 1997 contest the winning stock (CFL) had a volatility of only 18% and a beta close to one which would appear to negate our theoretical claims that a highly volatility and/or negative beta security is required to win the contest. However, note that our model only provides *stochastic predictions* and there is always a chance that thee coin comes up "tails". More importantly, the losing stock in the 1997 contest (GNU) actually had the highest volatility of 78.7% which is consistent with our theory - namely that a high volatility stock provides the best odds of coming in first, or last. If the stock does not win, it is quite likely to be the worst performer. The high volatility places the probable outcomes on the extreme of the distribution and therefore one of two possible events occur. In 1998, once again, the highest volatility stock did not win the contest, but its (ATY)'s beta was in fact negative. More importantly, the two losing stocks (BWR and ECX) had the highest volatility of 86.9% and 57.4% respectively. Once again, volatility placed the outcome in the extremes. We emphasize, from a methodological point of view, that our volatility and beta was estimated using 12 months of data prior to the contest. Thus, the volatility of BWR and ECX in 1997 – on the eve of the 1998 contest – was 86.9% and

57.4%. In fact, despite coming in dead last, they were both rational choices and excellent candidates to win such a contest.

 In 1999, the winning stock (IFM) earned an astounding 459% return over the 12 months and was classified with a volatility (in 1998) of 85.4%. This volatility was the highest amongst the stocks selected for the 1999 contest, and in fact is precisely what theory would predict. The highest volatility (and low beta) stock placed first. In the year 2000, once again, theory was vindicated with DNR winning the contest and having a volatility much higher than the market, as well as the most negative beta amongst the stocks selected that year. Note that information about bid.com, the worst stock, was unavailable. This is likely due to its having ceased trading during the year which is yet another example of high volatility leading to extreme outcomes. In the year 2002, once again, the highest volatilities were at the extremes, and in the year 2003 the highest volatility and lowest (negative) beta placed first. As pointed out in the introduction, the winning stocks in 2003 (SNG) and in 2002 (KCH) were selected by the same individual, using a variant of the theoretical strategy described in the previous section. Of course, at the present time, the jury is still out on the winner for 2004, but FMI – the 2002 and 2003 winner's pick – is the highest volatility stock at 97.1%, albeit the beta is positive. But recall from Table #1 that the sigma effect is stronger than the correlation effect which is why it might make sense in practice to select a higher volatility even at the expense of a positive beta.

 In sum, consistent with our theoretical arguments, it seems that the most volatile and negatively correlated (or at least low beta) stocks were amongst the winning (or extreme losing) circle in any given year.

# **4. Conclusion and Implications for Portfolio Management**

Portfolio selection and wealth management is about good stock picking as well as balancing risk and return. But, in the context of investment contests, these qualities have little bearing on the outcome. And, though at first glance it might appear that maximizing

the probability of winning may seem like a reasonable and innocuous investment strategy<sup>7</sup> to apply in one's daily life, a deeper examination reveals its flaws. Interestingly, if the contest would require each contestant to select two stocks – and their returns were averaged – a large part of this perverse incentive would be eliminated.

The implications of our analysis go far beyond the strategies required to win a popular game. Anecdotal evidence suggests that individual investors observe and then mimic the selections made by so-called expert participants in such contests. In fact, although our sample is statistically small, we found that the average daily volume – on the Monday following the announcement in the G&M -- for the stocks selected by the previous year's winner, was 5 to 10 times the normal amount. These effects are consistent with an extensive and growing branch of research in the field of behavioral finance – see Odean and Barber (2003) or Rashes (2001) -- which documents persistent biases, mistakes and consumer susceptibility to noisy signals. It is therefore extremely important that professional fund managers, as well as the press who are involved in promoting and participating in these contests, emphasize the hazards of mimicking these stock picks with one's own investment portfolio.

# **Epilogue**

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As of the close of trading at the end of February 2004, Forbes Medi-Tech (stock symbol FMI) – which was selected in early 2004 as this year's pick by the 2002 and 2003 champion – has increased in value by more than 140% in the first two months of the year. The nearest competitor, Killiam Properties Inc. (KMP), has increased in value by 30%. If indeed this relative ranking is maintained for the remainder of the year, the G&M will have crowned an unheard of three-time winner. Yet, using the optimal strategy described in the paper, the chances of a *three-peat* performance is a mere  $(1/2)^3 = 1/8$ , which is close to 92 times greater than a naïve  $(1/9)^3 = 1/729$  and much less impressive than the press (or the winner) would be willing to admit.

 $<sup>7</sup>$  In fact, this strategy has been advocated on the portfolio level as a method of saving for retirement (i.e. maximize the</sup> probability of having a fixed dollar amount at age 65) by a number of authors, including Ho, Milevsky and Robinson (1996) in the pages of the *Canadian Investment Review*.

#### **Technical Appendix**

In this appendix we briefly summarize the analytics behind the *probability of* winning when there are a total of  $(2+1)$  competitors in the contest; two opponents plus ourselves. The computations can be generalized from 3 to  $n$  stocks by scaling-up the integral described below. We start by denoting the stock picks of our two competitors by the symbols  $S_1(\mu_1, \sigma_1, \rho_1)$  and  $S_2(\mu_2, \sigma_2, \rho_2)$ , where each stock is assumed to obey a LogNormal distribution (a.k.a. geometric Brownian motion) and parameterized by an expected return (drift), standard deviation (volatility), and correlation ( $\rho_{12} = \rho_{21}$ ) with the other security. These are standard assumptions in financial theory and underlie the foundations of the Capital Asset Pricing Model (CAPM). At this point, our model is obviously predicated on actually *knowing* our opponents' selections. In the body of the paper we discuss what to do when this knowledge is unavailable. Also, when dealing with n securities instead of  $(2+1)$ , each stock would be characterized by its relation to all other stocks, via a covariance matrix.

We must now select a third stock, denoted by  $S_3(\mu_3, \sigma_3, \rho_{31}, \rho_{32})$ , with the highest probability of beating the stocks selected by our two opponents. If we let  $\widetilde{S}_i, i = 1,2,3$ denote the random price of each of these stocks at the end of the contest (i.e. year-end) and then define  $X_i = \ln[\tilde{S}_i / S_i]$ ,  $i = 1,2,3$  to be the continuously compounded return, our problem boils down to computing the following:

$$
w := \Pr[X_3 > \max[X_1, X_2]] \,. \tag{eq.1}
$$

The variable (stock's investment return)  $X<sub>3</sub>$  will win the contest *if and only if* it beats the maximum of the two investment returns generated by the competitors. Note that by computing and working with logarithms of the actual stock price, we are transformed from a LogNormal environment into a world of normality. Of independent interest is the quantity  $R^* = E[\max[X_1, X_2, X_3]]$ , which is the investment return earned by the winner of the contest. It is relatively easy to show that  $R^* > \max[\mu_1, \mu_2, \mu_3]$ .

A multi-variate normal random variable is defined and characterized by its expected return and covariance matrix. Formally, let:

$$
\mathbf{C} := \begin{pmatrix} \sigma_1 \sigma_1 & \rho_{12} \sigma_1 \sigma_2 & \rho_{13} \sigma_1 \sigma_3 \\ \rho_{21} \sigma_2 \sigma_1 & \sigma_2 \sigma_2 & \rho_{23} \sigma_2 \sigma_3 \\ \rho_{31} \sigma_3 \sigma_1 & \rho_{32} \sigma_3 \sigma_2 & \sigma_3 \sigma_3 \end{pmatrix},
$$
 (eq.2)

where *C*[*i*, *j*] denotes the entry in the i'th row and j'th column of the covariance matrix. Note, of course, that  $\{\rho_{23} = \rho_{32}, \rho_{13} = \rho_{31}, \rho_{21} = \rho_{12}\}\$  and the distinct notation in equation (eq.2) is used for cosmetic purposes only. We now define the inverse covariance matrix **IC** :=  $inv(C)$ , and follow the same convention for the individual terms  $IC[i, j]$ . Also, we let  $D := |C|$ , which is the determinant of the inverse of the covariance matrix. It is relatively straightforward to confirm this in our 3-asset case

$$
D = -(\sigma_1^2 \sigma_2^2 \sigma_3^2 (\rho_{23}^2 + \rho_{12}^2 + \rho_{13}^2 - 2\rho_{12}\rho_{13}\rho_{23} - 1))^{-1}.
$$
 (eq.3)

Recall that the need for a positive semi-definite covariance matrix  $C$  imposes the algebraic condition of *D* > 0 which then translates into an admissible range of correlation values for the set { $\rho_{31}, \rho_{32}$ }. For example, when the standard deviations are all arbitrarily set to:  $\sigma_i = 1$ ,  $i = 1,2,3$ , and the correlation between the competitors' stocks is set to  $\rho_{12} = 0$ , the determinant is  $D = 1/(2 \rho^2 - 1)$ , from equation (eq.3), and the positive semi-definite condition imposed on the selected correlation collapses to the region  $\{-1/\sqrt{2} < \rho < 1/\sqrt{2}\}$ when  $\rho := \rho_{31} = \rho_{32}$ . Thus, caution is warranted when 'solving' for the best parameters that blindly maximize the probability of winning in equation (eq.1), since they may inadvertently fall in a region that is unacceptable from a covariance matrix perspective. Finally, we define the function:

$$
Q(x_1, x_2, x_3 | \mu_1, \mu_2, \mu_2, \sigma_1, \sigma_2, \sigma_3, \rho_{12}, \rho_{31}, \rho_{32})
$$
  
= 
$$
\sum_{i=1}^{3} \sum_{j=1}^{3} (x_i - \mu_i) IC[i, j](x_j - \mu_j)
$$
 (eq.4)

Equation (eq.4) is the usual quadratic form which is the basis of the ubiquitous normal distribution. Thus, for example, when all three correlations { $\rho_{ij}$ } and expected return { $\mu_i$ } parameters are set equal to zero and all three standard deviations  $\{\sigma_i\}$  are set equal to one, equation (eq.4) collapses to:  $Q(x_1, x_2x_3) = x_1^2 + x_2^2 + x_3^3$ 2 2  $Q(x_1, x_2x_3) = x_1^2 + x_2^2 + x_3^3$ .

With this notation in hand, the tri-variate probability density function (pdf) for the normal distribution can be written as:

$$
f(x_1, x_2, x_3) = \frac{\sqrt{D}}{(2\pi)^{3/2}} \exp\left\{-\frac{1}{2}Q(x_1, x_2, x_3)\right\},
$$
 (eq.5)

where the function  $Q(x_1, x_2, x_3)$  in the exponent suppresses the parameters for notational simplicity. The probability the random return  $\{X_3\}$  will beat the better of the two random returns  $\{X_1, X_2\}$  can be computed by integrating the pdf in equation (eq.5) over the region in which  $x_3$  >  $max[x_1, x_2]$ , which can be expressed in integral form as:

$$
w = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{\max[x_1, x_2]} f(x_1, x_2, x_3) dx_3 dx_1 dx_2.
$$
 (eq.6)

Finally, using geometric arguments and visualizing the region of interest in  $\mathbb{R}^3$ , the probability of winning the contest, w, can be re-written as:

$$
w = \int_{-\infty}^{\infty} \int_{-\infty}^{x_2} \int_{x_2}^{\infty} f(x_1, x_2, x_3) dx_3 dx_1 dx_2 + \int_{-\infty}^{\infty} \int_{x_2}^{\infty} \int_{x_1}^{\infty} f(x_1, x_2, x_3) dx_3 dx_1 dx_2 \qquad \text{(eq.7)}
$$

Equation (eq.7) is the basis of the calculations described in the body of the paper. It can be used in a number of interesting ways. First, assuming a pre-determined set of parameters – estimated using historical performance data -- for the opponents' selections  $\{\mu_1,\mu_2,\sigma_1,\sigma_2,\rho_{12}\}\$ , we can solve for the optimum combination of  $\{\mu_3,\sigma_3,\rho_{31},\rho_{32}\}\$  which maximizes w in equation (eq.7), taking into account the restriction imposed by  $D > 0$  and possibly some (realistic) limits on  $\mu_3$ . We can then search amongst the available securities in the market (TSX, contest) that exhibit statistical behavior closest to the optimum parameters. Alternatively, we can start by estimating the { $\mu_3, \sigma_3, \rho_{31}, \rho_{32}$ } parameters for each and every available security in the market, and then 'plugging' each estimated set into equation (eq.7) and then finally picking the stock (i.e. parameter set) with the highest integral value (i.e. the probability of winning.) Of course, in a contest with more than  $(2+1)$  participants, the bounds of integration, as well as the precise integrand in equation

(eq.7) will have to be modified accordingly. In practice, for those who are interested in implementing this procedure in higher dimensions, we recommend using a symbolic computational language such as Maple – see Monagan, Geddes, Labahan and Vorkoetter (1996) for a description of the software -- which is what we used to evaluate equation (eq.7).

Note that in the actual paper we have assumed the mean return parameters  $\{\mu_1, \mu_2, \mu_3\}$  are all set to zero, thus our methodology remains agnostic about the ability to estimate the expected return for any given security over the short length of the contest. However, we stress that *even* if we were to arbitrarily plug-in a particular value for  $\{\mu\}$ into equation (eq.7), the volatility and correlation structure would have a much greater influence on the probability of winning.

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